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W. G. Durbin

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1. Theory

The correction, due to kinetic heating, to be applied to the indicated reading of an aircraft resistance thermometer is given by the equation

$$\Delta T_K = K \frac{V^2}{2JC_p} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

where ΔT_K is the temperature correction in °C appropriate to a recovery factor K, V is the true airspeed in cm sec^{-1}, J is the mechanical equivalent of heat (4.18×10^7 erg cal^{-1}) and C_p is the specific heat of air at constant pressure (0.24 cal gm^{-1} $°C^{-1}$). (1). The recovery factor, K, represents the proportion of the kinetic energy of the air that is given up to the thermometer and theoretically lies between 0 and 1. If the quoted values of J and C_p and the conversion factor from cm sec^{-1} to kt be substituted in equation (1), it becomes

$$\Delta T_K = 1.322\, K \left(\frac{V}{100}\right)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

where V is now in kt.

The recovery factor depends on the design of the thermometer, the position in which it is mounted on the aircraft and, to some extent, the height at which the aircraft is flying. The quantity 1.322K is called the "Speed Correction Coefficient". It is usually denoted by α and the method of obtaining it is described in reference (2).

An airspeed indicator does not, in general, give readings of true airspeed because the density of the air at the level of flight is usually different from that obtaining when the instrument was calibrated. In the absence of any other errors the airspeed indicator will give readings of what is called the "Equivalent Airspeed". The equivalent airspeed is related to the true airspeed by an equation whose first approximation is

$$V^2 = V_E^2 \left(\frac{\rho_0}{\rho}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

where V and V_E are the true and equivalent airspeeds and ρ_0 and ρ are the densities of air at the base of the I.C.A.N. atmosphere and at the level of flight respectively.

/Equation

Equation 3 may be rewritten as

$$V^2 = V_E^2 \frac{P_o T'}{T_o' P}$$ where P_o, T_o', P and T' are the air pressures

and temperatures (°K) respectively corresponding to ρ_o and ρ. Substituting 1013.2 mb for P_o and 288°K for T_o', these representing conditions at the base of the I.C.A.N. atmosphere, the above equation becomes

$$V^2 = 3.518 V_E^2 \frac{T'}{P}$$ where T' is the true air temperature in °K and

P is the true air pressure in mb at the level of flight, or,

$$V^2 = 3.518\ V_E^2 \frac{(T + 273)}{P}$$

where T is the true air temperature in °C.

Substituting this value for V^2, equation (2) becomes

$$\Delta T_K = 4.65K \frac{(T + 273)}{P} \left(\frac{V_E}{100}\right)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

The true air temperature, T, is, of course, not known and must be written as $T = T_i - \Delta T_K$ where T_i is the indicated temperature.

Equation (5) therefore becomes

$$\Delta T_K = \frac{4.65K(T_i - \Delta T_K + 273)}{P} \frac{(V_E)^2}{(100)} \quad \ldots\ldots\ldots (6)$$

Rearranging, equation (6) reduces to

$$\Delta T_K = (T_i + 273) \frac{K\phi}{(1 + K\phi)} \quad \ldots\ldots\ldots (7)$$

where ϕ is $\frac{4.65\ V_E^2}{100^2 P}$.

In the particular case for which K = 1, equation (7) becomes

$$\Delta T_{K=1} = (T_i + 273) \frac{\phi}{(1 + \phi)} \quad \ldots\ldots\ldots (8)$$

so that $\frac{\Delta T_K}{\Delta T_{K=1}} = \frac{K(1 + \phi)}{(1 + K\phi)} = \frac{K(1 + 1/\phi)}{(K + 1/\phi)}$

or $\Delta T_K = f(K)\ \Delta T_{K=1} \quad \ldots\ldots\ldots\ldots\ldots (9)$

where $f(K) = \frac{K(1 + 1/\phi)}{(K + 1/\phi)}$

Equation (9) is the basic equation used for the construction of the temperature - correction diagram.

2. Construction of the diagram

The principle behind the diagram is to draw isopleths of $\Delta T_{K=1}$, f(K) and the product of these. The first step in the preparation of the diagram was to derive isopleths of $\Delta T_{K=1}$ for the limits of indicated height, airspeed and temperature required. These limits were chosen to be from -2,000 ft to +53,000 ft on the I.C.A.N. scale, from 60 kt to 500 kt and from -67°C to +35°C respectively. The maximum indicated airspeed that can be catered for

falls off with increasing height but the diagram incorporates all conditions of height, airspeed and temperature likely to be encountered by aircraft whose performances are not significantly greater than those of conventional Canberra aircraft.

From equation (8),

$$\Delta T_{K=1} = (T_i + 273)\frac{1}{(1 + 1/\phi)} \quad \text{and} \quad \phi = \frac{4.65 V_E^2}{100^2 P}$$

The first isopleths plotted were those for constant values of the indicated airspeed V_i, in steps of 5 kt, on a graph having $1 + 1/\phi$ along the ordinate and P along the abscissa. The indicated airspeed differs from the equivalent airspeed due to calibration compressibility error and position error. Details of these errors are given elsewhere (e.g., appendix B of reference (2)). It suffices here to say that the former error increases with height and airspeed in a manner which can be determined exactly by theory. It is independent of the aeroplane and can therefore be allowed for in these isopleths. Position error, however, although generally comparatively small, depends on the design and position on the aircraft of the pitot tube and static vent and cannot be allowed for.

Allowance for compressibility was therefore made and the curves were plotted on paper which was logarithmic both along the ordinate and the abscissa. This has the advantage of greatly reducing the crowding of the curves compared with that which would be obtained by plotting on linear graph paper. It also has the advantage that, since the pressure - height relationship is logarithmic, the pressures along the abscissa can be converted to I.C.A.N. heights which are conveniently spaced. These isopleths therefore allow values of $1 + 1/\phi$ to be obtained for different values of indicated height and airspeed.

The second step in the construction of the diagram was to draw isopleths of $\Delta T_{K=1}$, this being obtained from equation (8) as the quotient $\frac{T_i + 273}{1 + 1/\phi}$. These were drawn on a second sheet of graph paper, this time logarithmic along the ordinate and linear along the abscissa. Values of $1 + 1/\phi$, between the same limits as for the first graph, were plotted along the ordinate and values of the indicated temperature, between +35°C and -67°C were plotted along the abscissa. From the values of $1 + 1/\phi$ obtained from the first graph and the values of the indicated temperature, this second graph allows values of $\Delta T_{K=1}$ to be obtained.

The third step was to superimpose isopleths of f(K) on the second graph. Values of $f(K)$ $\left(= \frac{K(1 + 1/\phi)}{(K + 1/\phi)} \right)$ were computed for the same limits of $1 + 1/\phi$ as before and for values of K between 0.5 and 1.5. For these isopleths, values of $1 + 1/\phi$ were again taken along the ordinate and values of K were taken along the abscissa. The value of K = 1.5 is greater than that theoretically possible (i.e., 1.0) but in cases where the mounting of the thermometer is such that the airspeed over it is greater than the speed of the aircraft, apparent values of K greater than 1.0 can result. To give one example, the mean value for all heights of the recovery factor for the navigator's thermometer on the Canberra aircraft of the Meteorological Research Flight was found to be 1.06. This corresponds to the value of 2.53 given for speed correction coefficient in reference (2). The value found at 40,000 ft, for the same thermometer, was 1.23, the corresponding value of the speed correction coefficient being 2.94.

The curves of f(K) were plotted in steps of 0.1 in K giving a total of 11 curves between the chosen limits of K. It is unnecessary to have more than this number since it is a simple matter to interpolate between successive curves and errors which might arise due to poor interpolation are negligible.

/The

The next step in the construction of the diagram was to draw isopleths of ΔT_K on the second graph. These were obtained by a gridding process as the product of the curves for $\Delta T_{K=1}$ and those for f(K) and were drawn in steps of 0.5°C for values of ΔT_K between 2°C and 40°C.

The final step was to superimpose the first and second graphs, retaining a background graph and the completed diagram is shown in Fig. 1. From the manner of construction of the diagram it is unnecessary to know the ordinate values and these have been omitted. Furthermore, it is more suitable to retain a linear than a partly or fully logarithmic background graph and this has been done.

3. Use of the diagram

The full instructions for the use of the diagram are as follows:-

(1) Obtain the point on the diagram corresponding to the indicated values of height (corrected for instrument error, static pressure error and converted to an altimeter setting of 1013.2 mb) and airspeed (corrected for instrument error and static pressure error).

(2) Move from the point obtained in (1) parallel to the abscissa to the value of the indicated temperature (corrected for instrument error) and obtain a second point.

(3) Move from this second point parallel to the pecked lines to a third point at the junction with the appropriate curve for f(K), i.e., the f(K) curve corresponding to the required value of K.

(4) At this third point read off the required temperature correction, interpolating between the curves for ΔT.

4. Discussion of the diagram

Although first impressions of the diagram may be that it is complicated and difficult to use, this is really not so and it should not take the average person long to become proficient in its use.

Theoretically it gives the temperature correction exactly and this represents an advantage over the method previously used at the Meteorological Research Flight and probably used in modified forms by many other establishments. This previous method uses the equation

$$\Delta T = \frac{3.5344\ \alpha\ (T + 273)}{P} \frac{V_E^2}{(100)^2}$$ where α is the speed

correction coefficient. Tables of $3.5344\ \alpha\ (T + 273)$ were prepared for a range of values of T and for each required value of α. It has been usual to use, as the value of α for an aircraft, the mean of the values obtained at different levels by the method of speed runs. While this is probably not seriously in error, it does involve the assumption of a speed correction coefficient which is invariant with height. This is contrary to experience with both the Meteorological Office flat plate thermometer and most commercial types of aircraft thermometer, even though different experimenters have disagreed on the manner in which the speed correction coefficient does vary with height. In computing the temperature correction by this method it is first assumed that the indicated temperature is a first approximation to the true temperature and a value of $3.5344\ \alpha\ (T + 273)$ may be obtained from the appropriate table. After allowing for instrument and position error corrections, the indicated airspeed V_i has to be converted to the equivalent airspeed V_E and a first approximation of ΔT is obtained by a slide rule operation. A second approximation to the true temperature T is then obtained by subtracting the first approximation to ΔT from the indicated temperature and the process is then repeated. This gives a second approximation to ΔT which differs from the true value of ΔT by a much smaller amount. In practice, this second approximation may be taken as the true value.

/The

The temperature - correction diagram makes allowance for variations in the recovery factor, K, and renders the computation of tables unnecessary. Compressibility errors are allowed for in the airspeed isopleths, again avoiding the need for a correction table. No slide-rule computations are necessary and, since the diagram is theoretically exact, the temperature corrections obtained from it are as correct as the user can obtain them. The error in using the diagram is not likely to be more than about 0.1°C at the lower airspeeds or about 0.5°C at the higher airspeeds. Errors of these magnitudes are, however, equally likely by the method which involves using the slide rule.

The absolute accuracy of the obtained temperature correction depends, of course, on the validity of the underlying theory and on the reliability of the airspeed indicator, altimeter and temperature indicator. Furthermore, values of the recovery factor, K, obtained from pairs of speed runs, carried out at different times but under apparently the same conditions, usually exhibit a large scatter which increases with height. Uncertainties in the accuracy of the measuring instruments and in the value of the recovery factor lead to probable temperature errors which are substantially larger than those likely to result from using the diagram. For example, the probable error of the flat plate thermometer fitted to the Canberra aircraft of the Meteorological Research Flight was found to vary at 40,000 ft from about 0.4°C at 200 kt to about 2°C at 450 kt.

It is possible to produce temperature-correction diagrams of the type presented to include any aircraft performance characteristics. Such diagrams could be presented on an enlarged scale to make for greater accuracy in reading them.

It is also possible to devise an instrument based on this diagram. Such an instrument would use sliding scales for parallel movements and would render some of the isopleths unnecessary. It is hoped to produce a prototype computer in the near future.

5. Acknowledgements

The work of devising this diagram was undertaken following a suggestion by Mr. F. H. Ludlam, during a joint M.R.F. - Imperial College investigation, that for slow speed aircraft an approximation to the temperature correction might be obtained diagrammatically.

The author is grateful to Mr. M. P. Gladstone who assisted in the computations while attached to Meteorological Research Flight as a vacation student.

6. References

(1) Meteorological Air Observer's Handbook, M.O. 470, p.15.

(2) Durbin, W.G. 1955 Some further determinations made by the M.R.F. of the speed correction coefficients of aircraft thermometers. Meteorological Research Papers, Air Ministry, London, No. 930.

I.C.A.N. HEIGHT (THOUSANDS OF FEET)

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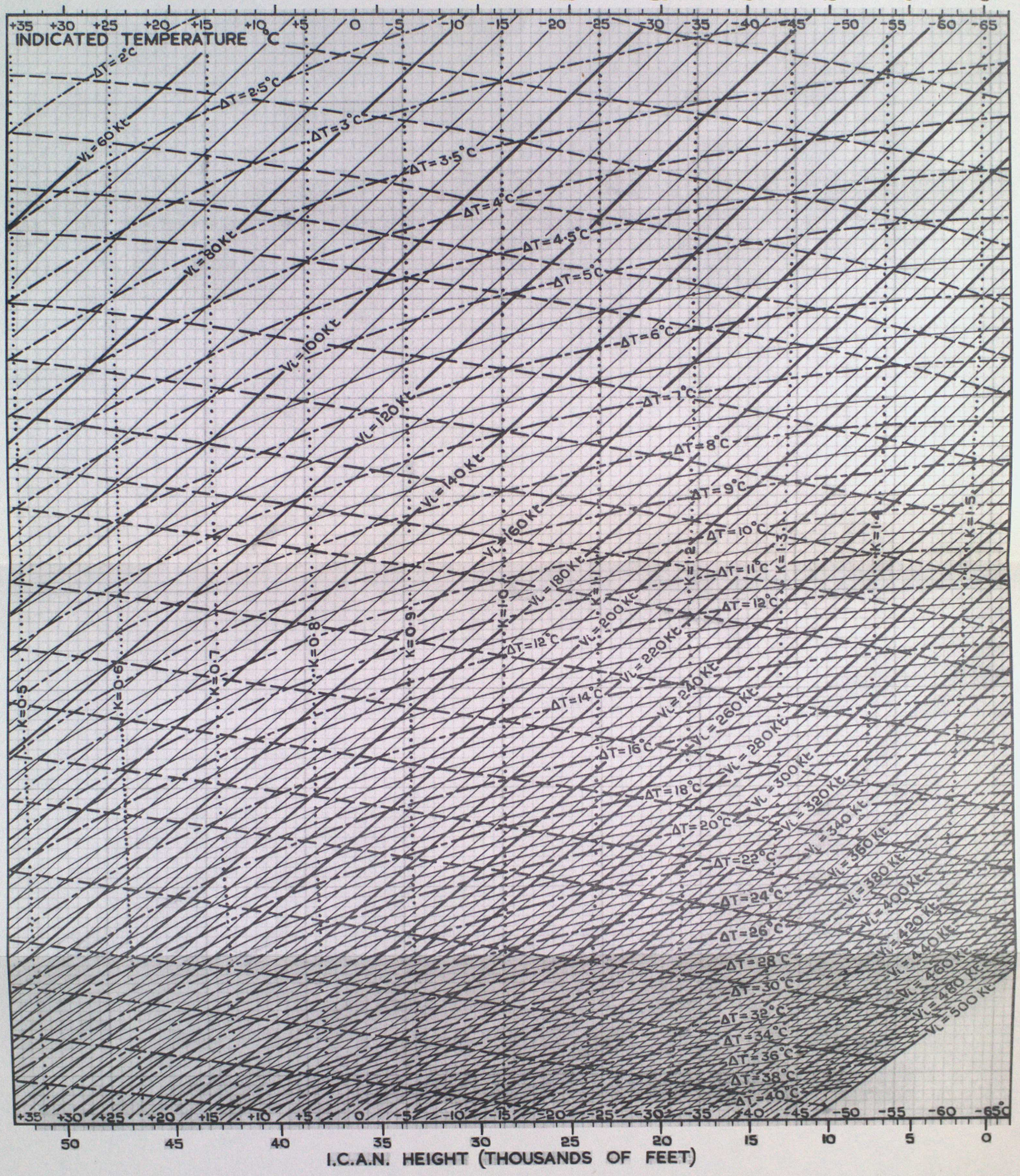

I.C.A.N. HEIGHT (THOUSANDS OF FEET)

TEMPERATURE–CORRECTION DIAGRAM FOR AIRCRAFT THERMOMETERS.